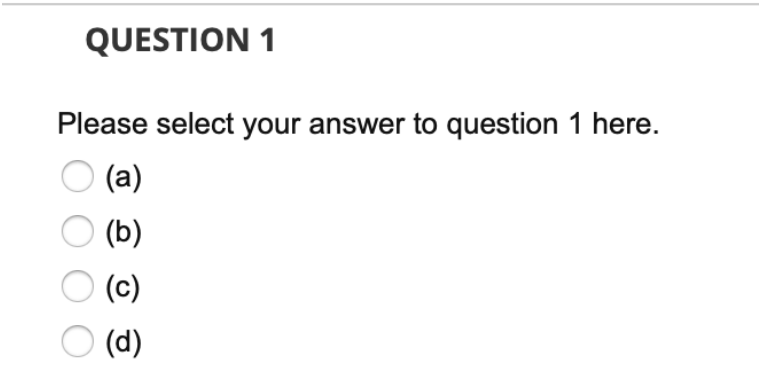
Math50004 Multivariable Calculus Quiz 1

Instructions

This quiz contains ten multiple choice questions worth 2 marks each. Please record your answers on the Blackboard quiz titled 'Multiple Choice Quiz 1.'

You may find it useful to note down the letter corresponding to your answer for each question as you go along. Below is a snippet of the Blackboard quiz:



You can save your progress and return to the quiz at any time within the 24 hour period. Once you are happy with all your answers please press 'save and submit' (only one submission is allowed). **The quiz will not auto-submit so please remember to press the submit button. The quiz is open for 24 hours from 9am on Monday 25th November.**

The questions will begin on the next page.

Let S be an open surface described by

$$z^2 = 25 - x^2 - y^2, \quad 3 \leq z \leq 4. \tag{1}$$

Let

$$\mathbf{A} = yz^3\,\hat{\mathbf{i}} + x^2y\,\hat{\mathbf{j}} + y^2z\,\hat{\mathbf{k}}. \tag{2}$$

During this quiz, we will evaluate the integral

$$I = \int_S (\mathrm{curl}\,\mathbf{A}) \cdot \hat{\mathbf{n}}\,\mathrm{d}S, \tag{3}$$

where $\hat{\mathbf{n}}$ is the unit normal to S with $\hat{\mathbf{n}} \cdot \hat{\mathbf{k}} < 0$.

We will evaluate the integral I in two different ways:

- By projecting S onto a plane and using the projection theorem
- By applying Stokes theorem and considering appropriate path integrals

Each of the ten questions is worth 2 marks.

Questions 1-5: Projection theorem

1. Calculate $(\mathrm{curl}\mathbf{A}) \cdot \hat{\mathbf{n}}$.

   (a) $\dfrac{3y^2z + z^3}{\sqrt{2}}$

   (b) $\dfrac{1}{5}(4xyz + 3y^2z^2 - z^4)$

   (c) $-\dfrac{1}{5}(4xyz + 3y^2z^2 - z^4)$

   (d) $-\dfrac{1}{5}(3y^2z^2 - z^4)$

2. Let Σ_z denote the projection of S onto the plane $z = 0$. What is the shape of Σ_z?

   (a) circular

   (b) elliptical

   (c) rectangular

   (d) annular

3. Using the projection theorem, we can write

(a) $I = -\int_{\Sigma_z} \left(4xy + 3y^2(25 - x^2 - y^2)^{1/2} - (25 - x^2 - y^2)^{3/2}\right) \, \mathrm{d}\Sigma_z$

(b) $I = -\int_{\Sigma_z} (4xy) \, \mathrm{d}\Sigma_z$

(c) $I = \int_{\Sigma_z} (4xy) \, \mathrm{d}\Sigma_z$

(d) $I = -\int_{\Sigma_z} \left(4xy + 3y^2(9 - x^2 - y^2)^{1/2} - (9 - x^2 - y^2)^{3/2}\right) \, \mathrm{d}\Sigma_z$

4. Using polar coordinates with the parameterisation $x = r\cos\theta$ and $y = r\sin\theta$, and an appropriate range for r and θ, the integral I can be written as which of the following?

(a) $-3\pi \int_3^4 r^3(25 - r^2)^{1/2} \, \mathrm{d}r + 2\pi \int_3^4 r(25 - r^2)^{3/2} \, \mathrm{d}r$

(b) $3\pi \int_3^4 r^3(25 - r^2)^{1/2} \, \mathrm{d}r - 2\pi \int_3^4 r(25 - r^2)^{3/2} \, \mathrm{d}r$

(c) $-3\pi \int_3^4 r^2(25 - r^2)^{1/2} \, \mathrm{d}r + 2\pi \int_3^4 (25 - r^2)^{3/2} \, \mathrm{d}r$

(d) $3\pi \int_0^5 r^3(25 - r^2)^{1/2} \, \mathrm{d}r - 2\pi \int_0^5 r(25 - r^2)^{3/2} \, \mathrm{d}r$

5. Hence evaluate I. (You may use integration software if you wish).

(a) -576π

(b) -144π

(c) 144π

(d) 576π

Questions continue on the next page.

Questions 6-10: Stokes theorem

6. According to Stokes theorem, the integral I defined by (1)-(3) is equivalent to

$$\oint_{\gamma_1} \mathbf{A} \cdot \mathrm{d}\mathbf{r} + \oint_{\gamma_2} \mathbf{A} \cdot \mathrm{d}\mathbf{r}. \tag{4}$$

Suppose γ_1 represents the **lower** boundary (i.e. that corresponding to the smaller value of z). In what directions are γ_1, γ_2 traversed, as viewed from above?

(a) γ_1 anti-clockwise and γ_2 clockwise

(b) γ_1 clockwise and γ_2 anti-clockwise

(c) γ_1 anti-clockwise and γ_2 anti-clockwise

(d) γ_1 clockwise and γ_2 clockwise

7. γ_1 can be described as the boundary

(a) $x^2 + y^2 = 25, z = 0$

(b) $x^2 + y^2 \leq 25, z = 0$

(c) $x^2 + y^2 = 16, z = 3$

(d) $x^2 + y^2 = 9, z = 4$

8. Which of the following integrals **does not** describe the contribution around γ_2?

(a) $I = \oint_{\gamma_2} yz^3 \,\mathrm{d}x + x^2 y \,\mathrm{d}y + y^2 z \,\mathrm{d}z$

(b) $I = \oint_{\gamma_2} yz^3 \,\mathrm{d}x + x^2 y \,\mathrm{d}y$

(c) $I = \oint_{\gamma_2} x^2 y \,\mathrm{d}y$

(d) $I = \oint_{\gamma_2} 64y \,\mathrm{d}x + x^2 y \,\mathrm{d}y$

9. Consider the parameterisation $x = 3\cos\theta$ and $y = 3\sin\theta$ for $0 \leq \theta \leq 2\pi$. Which of the following integrals describes the contribution to I along γ_2?

(a) $I = 81\displaystyle\int_{2\pi}^{0} \cos^3\theta \sin\theta \, \mathrm{d}\theta$

(b) $I = -576\displaystyle\int_{0}^{2\pi} \sin^2\theta \, \mathrm{d}\theta + 81\int_{0}^{2\pi} \cos^3\theta \sin\theta \, \mathrm{d}\theta$

(c) $I = -576\displaystyle\int_{2\pi}^{0} \sin^2\theta \, \mathrm{d}\theta + 81\int_{2\pi}^{0} \cos^3\theta \sin\theta \, \mathrm{d}\theta$

(d) $I = -192\displaystyle\int_{2\pi}^{0} \sin^2\theta \, \mathrm{d}\theta + 81\int_{2\pi}^{0} \cos^3\theta \sin\theta \, \mathrm{d}\theta$

10. Calculate I by summing the contributions along γ_1 and γ_2.

(a) 144π

(b) -576π

(c) 576π

(d) -144π

END OF QUIZ

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